The decay of unstable k-strings in SU(N) gauge theories at zero and finite temperature

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Abstract

Sources in higher representations of $SU(N)$ gauge theory at $T = 0$ couple with apparently stable strings with tensions depending on the specific representation rather than on its $N-$ality. Similarly at the deconfining temperature these sources carry their own representation-dependent critical exponents. It is pointed out that in some instances one can evaluate exactly these exponents by fully exploiting the correspondence between the 2+1 dimensional critical gauge theory and the $2d$ conformal field theory in the same universality class. The emerging functional form of the Polyakov-line correlators suggests a similar form for Wilson loops in higher representations which helps in understanding the behaviour of unstable strings at $T = 0$. A generalised Wilson loop in which along part of its trajectory a source is converted in a gauge invariant way into higher representations with same $N-$ality could be used as a tool to estimate the decay scale of the unstable strings.

1 Introduction

One of the most fascinating aspects of numerical experiments in lattice gauge theory is the possibility to make many controlled changes to explore the response of the strong interaction dynamics. In particular, we can vary the quark masses, the number N of colours and even remove the sea quarks. In this way one is led to study pure $SU(N)$ gauge theory to probe the main properties of the confining vacuum.

In this context, the linear rising of the static potential between a pair of quarks in the fundamental representation is well described by a thin flux tube, or string, joining the two quarks. Excited colour sources, *i.e.* sources in a

higher representation $\mathcal{R}$ of the gauge group, behave at intermediate distances in a similar way, giving rise to the formation of a confining string with a string tension $\sigma_{\mathcal{R}}$. However most strings of this kind are expected to be unstable: the long distance properties of the string attached to a source in a representation $\mathcal{R}$ built up of j copies of the fundamental representation should depend only on its $N-$ality $k_{\mathcal{R}} \equiv j \pmod{N}$ because all representations with same k can be converted into each other by the emission of a proper number of soft gluons. As a consequence, the heavier $\mathcal{R}-$strings are expected to decay into the string with smallest string tension within the same N-ality class, called $k-$string. For a recent discussion on this subject see [1].

General heuristic arguments suggest that stable $k-$strings belong to the anti-symmetric representation with k quarks, as it has been supported by exact results in various approaches to $\mathcal{N}=1$ supersymmetric $SU(N)$ gauge theories [2, 3, 4] as well as by lattice calculations in some $SU(N)$ pure gauge models [5, 6, 7, 8].

In this paper we address the question of stability of strings attached to sources in different representations with the same $N-$ality. Although most numerical experiments based on large Wilson loops [9, 10, 11, 12, 13] seem to defy the above theoretical arguments, yielding apparently stable string tensions which depend on $\mathcal{R}$ rather than on its $N-$ality, this kind of behaviour has been fully understood, at least in the case of the adjoint string. Since the $N-$ality of the adjoint representation is zero, the static potential of two adjoint sources is expected to level off at large separations. Correspondingly, the associated string should decay into a pair of bound sates of a static adjoint colour source and a gluon field, sometimes called glue-lump in the literature. Knowing the mass of the glue-lumps one can even evaluate the scale r_{adj} at which the adjoint string breaks [10].

The lack of any sign of adjoint string breaking in the above-mentioned studies, while measuring the static potential from Wilson loops only, indicates that such an operator has a poor overlap with the true ground state. This fact has been directly demonstrated in $2+1$ $SU(2)$ gauge theory [14] where, using a variance reduction algorithm allowing to detect signals down to 10^{-40}, it has been clearly observed a rectangular Wilson loop $W(r > r_{adj}, t)$ changing sharply its slope as a function of t from that associated to the unbroken string (area-law decay) to that of the broken-string state (perimeter-law decay) at a distance much longer than the adjoint string breaking scale r_{adj}.

Alternatively, one can enlarge the basis of the operators used to extract the adjoint potential in order to find a better overlap to the true ground state, following a multichannel method originally advocated in [15]. Indeed adjoint sources, contrarily to what happens in the case of fundamental representation, can form gauge invariant open Wilson lines, like those depicted in Fig.1 a and b, having a good overlap with the two glue-lump state. In this way a rather abrupt crossover between string-like and broken string states has been clearly seen at the expected distance r_{adj} both in $2+1$ [16] and $3+1$ [17] $SU(2)$ gauge models. The same multichannel method allowed to observe breaking of fundamental string in various gauge theories coupled to dynamical matter fields

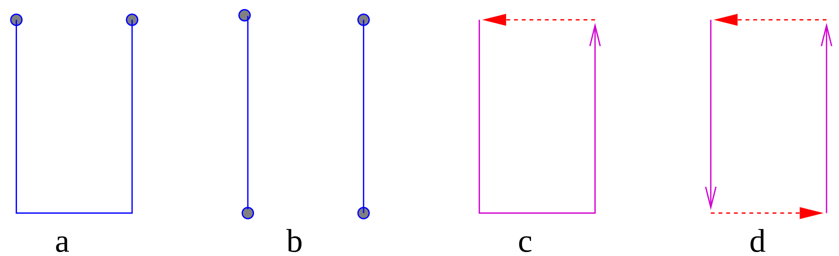

Figure 1: Glue-lump operators and mixed Wilson lines. The operators depicted in a and b describe adjoint Wilson lines decaying in the vacuum. They are used to extract the adjoint static potential in the multichannel method. The operators c and d, described in the text, generalise the glue-lump operators to the case of representations of non zero $N-$ality .

in 2+1 [18] and 3+1 dimensions [19], including QCD with two flavours [20]. Also in these cases ordinary Wilson loops do not show clear signs of string breaking even at sizes much larger than the expected breaking scale, except in a particularly simple case, the 2+1 $\mathbb{Z}_2$ gauge-Higgs model [22], where, using a variance reduction algorithm like in [14], fundamental string breaking has been convincingly demonstrated.

One of the results of the present work is the generalisation of the multichannel method to the decay of unstable strings of non-vanishing $N-$ality. To this aim we define in Sect.4 a new kind of gauge-invariant operators, the mixed Wilson loops, (see Fig.1 c and d) where along one or more segments of the closed path γ (the dashed lines in Fig.1) the static source carries the quantum numbers of an excited representation $\mathcal{R}$ of $N-$ality k, while in the remaining path (solid lines) the source lies in the stable, fully anti-symmetric representation. These operators have a good overlap with the stable $k-$string state and constitute the most obvious generalisation of the glue-lump operators drawn in Fig.1 a and b.

The above construction follows a general discussion of the expected overlap properties of ordinary Wilson loops in excited representation. This leads to a rough estimate of the decay scale of the unstable strings and offers a simple explanation of the apparent stability of these strings at $T = 0$.

The $T > 0$ situation is less problematic. According to a simple diagrammatic argument [23] the correlator of Polyakov lines is expected to have a good overlap with the true ground state. These correlators in non-fundamental representations have been studied in various instances [24, 25, 27, 28] and it has easily observed the screening of the adjoint representation, and the decay of excited representations of non vanishing triality in $SU(3)$ and even the decay of the symmetric 2-index representation into the anti-symmetric one in $SU(4)$ [29]. A difficulty of the $T > 0$ approach is to extract reliable estimates of the breaking or decay scales of unstable strings at $T = 0$.

A much more challenging problem emerges at the deconfining point: in gauge theories with continuous phase transition one obvious question concerns the critical behaviour of the Polyakov lines in arbitrary representations. Over the years,many studies have been dedicated to this subject [30, 31, 32, 33, 34, 35].

The surprising result is that sources of higher representations, according to various numerical experiments [31, 33, 34], correspond to different magnetisation exponents, one exponent for each representation. Universality arguments would, for such continuous phase transitions, place the finite-temperature $SU(N)$ gauge theory in the universality class of $\mathbb{Z}_N$ invariant spin model in one dimension less [36]. The spin operator is mapped into the Polyakov line in the fundamental representation. What about the Polyakov lines in higher representations of $SU(N)$? There appears to be no room for independent exponents for these higher representations from the point of view of the abelian spin system, since there is simply no obvious analogue of the non-abelian process of soft gluons emission, creating higher representations in the same $N-$ality class.

Actually a mean field approximation of the the effective $SU(2)$ Polyakov-line action at criticality in the $d \to \infty$ limit shows that the leading amplitudes of higher representations vanish at strong coupling, and the sub-leading exponents become dominant, thus each higher representation source carries its own critical exponent[34].

Here we reach a similar conclusion for $SU(3)$ and $SU(4)$ critical theories in $2+1$ dimensions starting from a different point of view: we find a map between the operator product expansion (OPE) of the Polyakov operators in the gauge theory and the corresponding spin operators in the two-dimensional conformal field theory which describes the associated spin system at criticality. As a result, we are able to conjecture, for instance, an exact value for the η exponent (or anomalous dimension) of the sources in the *symmetric* representation made with two quarks (Sect.2). The resulting functional form of the Polyakov-Polyakov correlator at criticality of this excited representation is the starting point of an Ansatz for the vacuum expectation value of the Wilson loop associated to sources in higher representations at $T = 0$ (Sect.3). Section 4 is dedicated to the construction of mixed Wilson loops and finally in Sect. 5 we draw some conclusions.

2 Polyakov loops at criticality

Consider a $d+1$ dimensional pure gauge theory undergoing a continuous deconfinement transition at the critical temperature T_c. The effective model describing the behaviour of Polyakov lines at finite T will be a $d-$dimensional spin model with a global symmetry group coinciding with the center of the gauge group. Svetitsky and Yaffe [36] (SY) were able to show that this effective model has only short-range interactions, then it follows from universality arguments that the spin model belongs in the same universality class of the original gauge model and the high temperature phase of the gauge theory is like the low temperature phase of the spin system.

It is clear that this SY conjecture, which has passed several numerical tests, becomes very predictive for $d = 2$, where, using the methods of conformal field theory (CFT), the critical behaviour can be determined exactly. For example, the critical properties of $2 + 1$ dimensional $SU(3)$ gauge theory at deconfinement coincide with those of the 3-state Potts model, as it has been checked in numerical simulations [33].

What is needed to fully exploit the predictive power of the SY conjecture is a mapping relating the physical observables of the gauge theory to the operators of the dimensionally reduced model, as first advocated in [37].

The correspondence between the Polyakov line in the fundamental representation f and the order parameter σ of the spin model is the first entry in this mapping:

$$\mathrm{tr}_f(U_{\vec{x}}) \ \sim \ \sigma(\vec{x}) \ , \tag{2.1}$$

$U_{\vec{x}}$ is the gauge group element associated to the closed path winding once around the periodic imaginary time direction intersecting the spatial plane at the point $\vec{x}$. The above equivalence is only valid in a weak sense, that is, when the left-hand-side of the equation is inserted in a correlation function of the gauge theory and the right-hand side in the corresponding correlator of the spin model.

It is now natural to ask what operators in the CFT correspond to Polyakov lines in higher representations. On the gauge side these can be obtained by a proper combination of products of Polyakov lines in the fundamental representation, using repeatedly the general property

$$\mathrm{tr}_{\mathcal{R}}(U)\,\mathrm{tr}_{\mathcal{S}}(U) = \sum_{\mathcal{R}' \in \mathcal{R} \otimes \mathcal{S}} \mathrm{tr}_{\mathcal{R}'}(U) \ , \tag{2.2}$$

valid for any pair of irreducible representations of an arbitrary group. In order to control the singularities in the correlator functions due to evaluation of local operators at the same point we may resort to the operator product expansion (OPE). The OPE of Polyakov operator in the fundamental representation can be written in the form

$$\mathrm{tr}_f(U_{\vec{x}})\ \mathrm{tr}_f(U_{\vec{y}}) = \sum_{\mathcal{R} \in f \otimes f} C_{\mathcal{R}}(|\vec{x} - \vec{y}|)\,\mathrm{tr}_{\mathcal{R}}(U_{(\vec{x}+\vec{y})/2}) + \dots \tag{2.3}$$

where the coefficients $C_{\mathcal{R}}(r)$ are suitable functions (they become powers of r at the critical point) and the dots represent the contribution of higher dimensional local operators. The important property of this OPE is that the local operators are classified according to the irreducible representations of G obtained by the decomposition of the direct product of the representations of the two local operators in the left-hand side.

On the CFT side we have a similar structure. The order parameter σ belongs to an irreducible representation $[\sigma]$ of the Virasoro algebra [38] and the local operators contributing to an OPE are classified according to the decomposition of the direct product of the Virasoro representations of the left-hand-side operators. This decomposition is known as fusion algebra and can be written

generically as

$$[\chi_i] \star [\chi_j] = c_{ij}^k [\chi_k] \tag{2.4}$$

where the integers c_{ij}^k are the fusion coefficients. In the case of three-state Potts model there is a finite number of representations that we list along with their scaling dimensions [1]

$$\begin{array}{llll} \mathbb{I} \text{ (identity)}; & \sigma,\, \sigma^+ \text{ (spin fields)}; & \epsilon \text{ (energy)}; & \psi, \psi^+ \\ x_{\mathbb{I}} = 0 & x_\sigma = \frac{2}{15} & x_\epsilon = \frac{4}{15} & x_\psi = \frac{4}{3} \end{array} \tag{2.5}$$

The fusion rules we need are

$$[\sigma] \star [\sigma] = [\sigma^+] + [\psi^+] \ ; \ \ [\sigma] \star [\sigma^+] = [\mathbb{I}] + [\epsilon] \ ; \ \ [\psi] \star [\epsilon] = [\sigma] \ . \tag{2.6}$$

Comparison of the first equation with the analogous one of the gauge side

$$\{3\} \otimes \{3\} = \{\bar{3}\} + \{6\} \ , \tag{2.7}$$

owing to the correspondence $\mathrm{tr}_{\bar{f}}(U_{\vec{x}}) \sim \sigma^+(\vec{x})$, yields a new entry of the gauge/CFT correspondence

$$\mathrm{tr}_{\{6\}}(U_{\vec{x}}) \ \sim \ \psi^+(\vec{x}) + c\, \sigma^+(\vec{x}) \ , \tag{2.8}$$

where there are no a priori reasons for the vanishing of the coefficient c. Hence the Polyakov-Polyakov critical correlator of the symmetric representation $\{6\}$ is expected to have the following general form in the thermodynamic limit

$$\langle \mathrm{tr}_{\{6\}}(U_{\vec{x}})\, \mathrm{tr}_{\{\bar{6}\}}(U_{\vec{y}}) \rangle_{SU(3)} = \frac{c_s}{r^{2x_\sigma}} + \frac{c_u}{r^{2x_\psi}} \ , \tag{2.9}$$

with $r = |\vec{x} - \vec{y}|$ and c_s, c_u suitable coefficients. Since $x_\sigma < x_\psi$, the second term drops off more rapidly than the first, thus at large distance this correlator behaves like that of the anti-symmetric representation $\{\bar{3}\}$ as expected also at zero temperature.

A similar reasoning can be applied to the second fusion rule (2.6), which is the general form relating the order parameter to the energy operator in spin systems. The gauge side of this equation is

$$\{3\} \otimes \{\bar{3}\} = \{1\} + \{8\} \tag{2.10}$$

which leads to the new entry

$$\mathrm{tr}_{adj}(U_{\vec{x}}) \ \sim \ a + \epsilon(\vec{x}) \ , \tag{2.11}$$

which is expected to be valid for any $SU(N)$ gauge theory undergoing a continuous phase transition. The constant a can be numerically evaluated[32] using the expected finite-size behaviour

$$\langle \mathrm{tr}_{adj}(U) \rangle = a + \frac{b}{L^{2-1/\nu}} \ , \tag{2.12}$$

[1] Actually the critical three-state Potts model is invariant under a larger algebra than that of Virasoro, the so-called $\mathcal{W}_3$ algebra, and the representations listed in (2.5) are irreducible representations of such a larger algebra. A complete list of these fusion rules can be found for instance in [39].

where L is the spatial size of the system and we used the general relation $x_\epsilon = d - 1/\nu$ relating the scaling dimension of the energy operator to the thermal exponent ν.

Finally the third fusion rule (2.6), combined with (2.11), can be interpreted as the CFT counterpart of the soft gluon emission which converts into each other stable ($\sim \sigma$) and unstable ($\sim \psi$) strings.

Another example of this gauge/CFT correspondence can be worked out in the 2+1 dimensional $SU(4)$ gauge model, where now the $2d$ CFT is the symmetric Ashkin-Teller model [40]. It describes two Ising models with spin fields $\sigma(\vec{x})$ and $\tau(\vec{x})$ with scaling dimensions $x_\sigma = x_\tau = \frac{1}{8}$ coupled through a local four spin interaction which depends on a coupling constant g.

The model is invariant under three different $\mathbb{Z}_2$ transformations:

$$\sigma \to -\sigma \ ; \quad \tau \to -\tau \ ; \quad \sigma \leftrightarrow \tau \ . \tag{2.13}$$

The first two correspond to symmetries that can be spontaneously broken by a thermal variation, leading to an order-disorder transition. Hence they should be associated to the center $\mathbb{Z}_4$ of the gauge group. The $\sigma \leftrightarrow \tau$ symmetry, on the contrary, cannot be spontaneously broken by a thermal variation and corresponds to charge conjugation. The first entry of the gauge/CFT correspondence can therefore be written in the form

$$\mathrm{tr}_{\{4\}}(U_{\vec{x}}) \ \sim \ \theta(\vec{x}) = e^{-i\pi/4} \ (\sigma(\vec{x}) + i\tau(\vec{x})) \ . \tag{2.14}$$

The phase $\pi/4$ comes from the observation that charge conjugation $\sigma \leftrightarrow \tau$ should correspond, on the gauge side, to complex conjugation; this gives the constraint

$$\theta(\vec{x}) \underset{\sigma \leftrightarrow \tau}{\longrightarrow} \theta^*(\vec{x}) \ , \tag{2.15}$$

which fixes such a phase factor.

The local operators which occur in the OPE of $\theta(\vec{x})\ \theta(\vec{y})$ or $\theta(\vec{x})\ \theta^*(\vec{y})$ are the so-called "ploarization" $\pi(\vec{x}) \in [\pi] = [\sigma] \star [\tau]$ and the "cross-over" operator $\rho(\vec{x}) \in [\rho] = [\sigma] \star [\sigma] - [\tau] * [\tau]$ as well as the energy ϵ and the identity.

The Ashkin-Teller model is not an isolated critical system like the three-state Potts model, but describes a line of fixed points depending on the coupling parameter g. While the spin fields retain along the fixed line their scaling dimensions, ϵ, π and ρ have $g-$dependent scaling dimensions. They obey, however, the simple relations

$$\frac{x_\pi}{x_\epsilon} = \frac{1}{4} \ , \quad x_\rho \, x_\epsilon = 1 \ . \tag{2.16}$$

Numerical experiments [41] indicate that the critical $SU(4)$ gauge theory is located near the four-state Potts model, corresponding to $x_\epsilon = \frac{1}{2}$ [40].

Let us consider the $SU(4)$ representations made with two quarks. We have

$$\{4\} \otimes \{4\} = \{6\} + \{10\} \ , \tag{2.17}$$

where the anti-symmetric representation $\{6\}$ is real, *i.e.* $[\mathrm{tr}_{\{6\}}(U)]^* = \mathrm{tr}_{\{6\}}(U)$, being the vector representation of $SO(6) \sim SU(4)$. On the CFT side, using (2.14) in the OPE of $\theta(\vec{x})\,\theta(\vec{y})$ yields at once

$$[\theta] \star [\theta] = [\pi] + [\rho] \ , \tag{2.18}$$

where $[\pi] = [\sigma] \star [\tau]$ is real and even under charge conjugation $\sigma \leftrightarrow \tau$, while $[\rho]$ is purely imaginary and odd. We have therefore two new entries for the $SU(4)$/CFT correspondence

$$\begin{aligned} \mathrm{tr}_{\{6\}}(U_{\vec{x}}) \ &\sim \ \pi(\vec{x}) \ , \\ \mathrm{tr}_{\{10\}}(U_{\vec{x}}) \ &\sim \ \rho(\vec{x}) + c\,\pi(\vec{x}) \ , \end{aligned} \tag{2.19}$$

where c must be different from zero, because $\mathrm{tr}_{\{10\}}(U)$ is not purely imaginary.

In analogy with Eq.(2.9) we can write

$$\langle \mathrm{tr}_{\{10\}}(U_{\vec{x}})\ \mathrm{tr}_{\{\bar{10}\}}(U_{\vec{y}})\rangle_{SU(4)} = \frac{c_s}{r^{x_\epsilon/2}} + \frac{c_u}{r^{2/x_\epsilon}} \ , \tag{2.20}$$

with both c_s and c_u are different from zero. This shows that the Polyakov-Polyakov correlator in the symmetric representation with $N-$ality 2, even if at short distance is controlled by the irrelevant operator ρ, at large distance behaves exactly like the anti-symmetric rep. with the same $N-$ality, as expected under physical grounds.

3 Decay of unstable strings at zero temperature

As mentioned in the introduction, the difficulty in observing string breaking or string decay with the Wilson loop seems to indicate nothing more than that it has a very small overlap with the broken-string or stable string state. Why? being a general phenomenon which occurs for any gauge group, including $\mathbb{Z}_2$, in pure gauge models as well as in models coupled to whatever kind of matter, it requires a general explanation which should not depend on detailed dynamical properties of the model. A simple explanation in the case of gauge models coupled with matter was proposed in [22], which now we enforce in the present case.

The general form of the Polyakov correlator in higher representations of $SU(N)$ found in Eq.s(2.9) and (2.20) suggests a simple Ansatz describing the asymptotic functional form of the vacuum expectation value of a large, rectangular, Wilson loop in a higher representation $\mathcal{R}$ coupled to an unstable string which should decay into a stable $k-$string [2]

$$\langle W_{\mathcal{R}}(r,t)\rangle \simeq c_u \exp[-2\mu_{\mathcal{R}}(r+t) - \sigma_{\mathcal{R}} rt] + c_s \exp[-2\mu'_{\mathcal{R}}(r+t) - \sigma_k rt] \tag{3.21}$$

[2] For sake of simplicity we neglect the $1/r$ term in the potential which accounts for the quantum fluctuations of the flux tube.

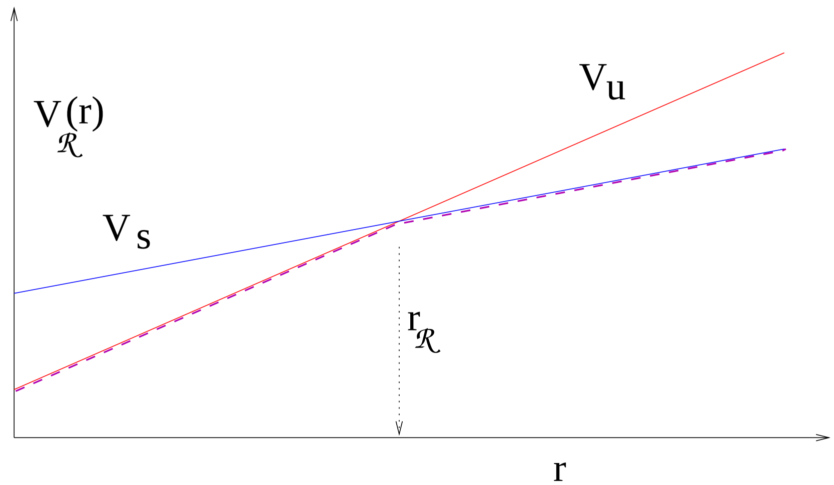

Figure 2: A schematic view of the static potential between sources belonging to an excited representation $\mathcal{R}$ (dashed line). V_u is the potential experienced at intermediate distances generated by the unstable string. V_s is the asymptotic behaviour, controlled by the string tension σ_k of the stable string in which the unstable string decays. $r_\mathcal{R}$ is the decay scale.

Similar proposals are described in [14, 1]. The first term describes the typical area-law decay produced at intermediate distances by the unstable string with tension $\sigma_\mathcal{R}$. The second term is instead the contribution expected by the stable $k-$string in which the $\mathcal{R}-$string decays. In the case of adjoint representation (zero $N-$ality) one has $\sigma_0 = 0$ and the perimeter term μ'_{adj} denotes the mass of the lowest glue-lump. Eq.(3.21) has to be understood as an asymptotic expansion which approximates $\langle W_\mathcal{R}(r,t)\rangle$ when $r, t > r_o$, where r_o may be interpreted as the scale where the confining string forms.

When t and r are sufficiently large, no matter how small c_s is, the above Ansatz implies that at long distances the stable string eventually prevails, since $\Delta\sigma \equiv \sigma_\mathcal{R} - \sigma_k > 0$, hence the first term drops off more rapidly than the second. With the emergence of this long-distance effect a closely related question comes in: at what distance the stable string shows up? it depends on the difference $\Delta\mu \equiv \mu'_\mathcal{R} - \mu_\mathcal{R}$. Since

$$-\frac{1}{t}\log\langle W_\mathcal{R}(r,t)\rangle = 2\mu'_\mathcal{R} + \sigma_k r - \frac{1}{t}\log\left[c_s + c_u e^{r2\Delta\mu} e^{-t(r\Delta\sigma - 2\Delta\mu)}\right] \ , \qquad (3.22)$$

we have (see Fig.2)

$$V_\mathcal{R}(r) = V_s(r) = 2\mu'_\mathcal{R} + \sigma_k r \ , \quad r > r_\mathcal{R} \ , \qquad (3.23)$$

where

$$V_\mathcal{R}(r) = -\lim_{t\to\infty}\frac{1}{t}\log\langle W_\mathcal{R}(r,t)\rangle \qquad (3.24)$$

is the static potential and $r_{\mathcal{R}}$ is the value of r which annihilates the exponent in Eq.(3.22)

$$r_{\mathcal{R}} = \frac{2\Delta\mu}{\Delta\sigma} \equiv 2\frac{\mu'_{\mathcal{R}} - \mu_{\mathcal{R}}}{\sigma_{\mathcal{R}} - \sigma_k} \; . \tag{3.25}$$

Creation of unstable strings requires $\mu'_{\mathcal{R}} > \mu_{\mathcal{R}}$ (see Fig.2). In the case of zero $N-$ality the above equation yields the usual estimate of the adjoint string breaking scale. The inclusion of the $\frac{1}{r}$ terms in this analysis does not modify substantially the numerical estimates. Notice that the mass $\mu_{\mathcal{R}}$ and $\mu'_{\mathcal{R}}$ are not UV finite because of the additive self-energy divergences (linear in 3+1 dimensions, logarithmic in 2+1 dimensions), which cannot be absorbed in a parameter of the theory. However, these divergences should cancel in their difference, hence $r_{\mathcal{R}}$ is a purely dynamical scale, defined for any non fully antisymmetric representation of $SU(N)$, which cannot be tuned by any bare parameter of the theory.

When r is less than the decay scale $r_{\mathcal{R}}$ Eq.(3.23) is no longer valid and is replaced by

$$V_{\mathcal{R}}(r) = V_u(r) = 2\mu_{\mathcal{R}} + \sigma_{\mathcal{R}} r \; , \quad r_o < r < r_{\mathcal{R}} \; . \tag{3.26}$$

Thus the Ansatz (3.21) describes the unstable string decay as a level crossing phenomenon, as observed in the adjoint string.

It is worth noting that, though c_u and c_s must be non-vanishing quantities, they do not contribute to $V_{\mathcal{R}}(r)$, whereas play a fundamental role in the possibility to observe string decay when both r and t are finite. It is easy to see that for $r < r_{\mathcal{R}}$ and t large enough the unstable string cannot decay, the reason being that $V_u(r) < V_s(r)$ in this range. In order to avoid the unphysical behaviour in which the decay is visible only in a finite interval of t, it is obvious that c_s cannot be too big. More precisely, we must assume

$$c_u \, e^{-2\mu_{\mathcal{R}}(r+t) - \sigma_{\mathcal{R}} rt} \geq c_s \, e^{-2\mu'_{\mathcal{R}}(r+t) - \sigma_k rt} \; , \; r_o \leq r \leq r_{\mathcal{R}} \; , \; t \geq r_o \; . \tag{3.27}$$

With the help of Eq.(3.25), this inequality can be recast into the form

$$\log \frac{c_s}{c_u} \leq \Delta\sigma \; [r_{\mathcal{R}}(r+t) - rt] \quad , \; r_o \leq r \leq r_{\mathcal{R}}, \; t \geq r_o \; . \tag{3.28}$$

To minimise the right-hand-side we put $r = t = r_o$ and get

$$\log \frac{c_s}{c_u} \leq \Delta\sigma \, r_o \, (2 r_{\mathcal{R}} - r_o) \quad . \tag{3.29}$$

Such an upper bound constitutes the main obstruction to observe string decay in the ordinary Wilson loop $W_{\mathcal{R}}$. Indeed, when $r > r_{\mathcal{R}}$ the distance $t_{\mathcal{R}}$ where the second term of the Ansatz (3.21) equals the first, then it makes possible to see the decay, is given by

$$t_{\mathcal{R}} \, (r - r_{\mathcal{R}}) = r \, r_{\mathcal{R}} - \frac{1}{\Delta\sigma} \log \frac{c_s}{c_u} \geq \; r_{\mathcal{R}} \, (r - 2r_o) + r_o^2 \quad , \tag{3.30}$$

which shows that $t_{\mathcal{R}} \gg r_{\mathcal{R}}$ unless $r \gg r_{\mathcal{R}}$, and we shall argue shortly that $r_{\mathcal{R}} > r_{adj}$. From a computational point of view it is very challenging to reach such length scales [3] in the measure of $\langle W_{\mathcal{R}}(r,t) \rangle$. This explains why unstable

[3] In 3+1 dimensional $SU(2)$ gauge model, for instance, $r_{adj} \sim 1.25$ fm [17].

string decay has not yet been observed at zero temperature.

The evaluation of $r_{\mathcal{R}}$ is problematic, because Eq.(3.25) implies an estimate of the self-energy $\mu'_{\mathcal{R}}$, which contributes to the sub-dominant term of the Ansatz (3.21), hence from a numerical point of view it is almost hopeless. We shall see that the mixed Wilson loop can be used as a simple tool to extract this quantity.

Waiting for a computational work which will provide us with this information, we now try to combine together some known facts in order to get a rough estimate of $r_{\mathcal{R}}$. Numerical data on unstable strings in 3+1 dimensional $SU(3)$ [11, 12] and $SU(2)$ [13] gauge theories seem to support Casimir scaling [42], which tells us that the static potential between sources in the representation $\mathcal{R}$ is proportional to that of fundamental sources according to

$$V_{\mathcal{R}}(r) \simeq \frac{\mathcal{C}_{\mathcal{R}}}{\mathcal{C}_f} V_f(r) \quad , \tag{3.31}$$

where $\mathcal{C}_{\mathcal{R}}$ is the quadratic Casimir operator of the representation $\mathcal{R}$. This implies

$$\sigma_{\mathcal{R}} \simeq \frac{\mathcal{C}_{\mathcal{R}}}{\mathcal{C}_f} \sigma \ , \quad \mu_{\mathcal{R}} \simeq \frac{\mathcal{C}_{\mathcal{R}}}{\mathcal{C}_f} \mu \ , \tag{3.32}$$

where σ and μ refer to the fundamental representation. Since the additive UV divergence of $\mu_{\mathcal{R}}$ is cancelled by that of $\mu'_{\mathcal{R}}$ in the difference $\Delta\mu$, it is quite natural to expect $\mu'_{\mathcal{R}}$ also proportional to $\mathcal{C}_{\mathcal{R}}$. However, only the unstable $\mathcal{R}-$strings have a non-vanishing μ', hence we assume

$$\mu'_{\mathcal{R}} \simeq \frac{\mathcal{C}_{\mathcal{R}}}{\mathcal{C}_{adj}} \ \mu'_{adj} \ , \tag{3.33}$$

where, as noted above, μ'_{adj} is the mass of the lowest glue-lump. Thus, combining Eq.s(3.25),(3.32) and (3.33) leads to

$$r_{\mathcal{R}} \simeq \frac{\mathcal{C}_{\mathcal{R}}}{\mathcal{C}_{\mathcal{R}} - \sigma_k/\sigma} \ r_{adj} \ , \tag{3.34}$$

hence $r_{\mathcal{R}} > r_{adj}$ for any unstable string coupled to a colour source with non-vanishing $N-$ality. For instance, in the $SU(2)$ case, the $j = \frac{3}{2}$ is unstable against decaying into the fundamental string with $j = \frac{1}{2}$, thus $r_{\frac{3}{2}} \simeq \frac{5}{4} r_{adj}$.

Notice that, unlike the unstable strings, the stable $k-$strings tension σ_k/σ does not obey Eq.(3.32). Numerical experiments [5, 6, 7, 8] as well as consistency of $N \to \infty$ limit [1] point to a sizable violation of the Casimir scaling and some works [5, 8] find good agreement with the so called sine law discovered in some supersymmetric gauge models[2, 3, 4].

Our rough estimate of the decay lengths of unstable strings were derived under a number of (strong) assumptions. A better definition, based on the multichannel method, will be described in the next section.

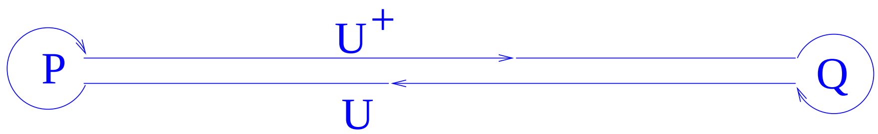

Figure 3: Generating operator of the gluelump, as described in Eq.(4.35).

4 Mixed Wilson loops

To warm up, let us consider the construction of a $SU(N)$ glue-lump operator which creates a glue-lump at the point $\vec{x}$ and annihilates it at the point $\vec{y}$.

The starting point is the gauge-invariant operator depicted in Fig. 3

$$P(\vec{x})^i_j \;\; U^\dagger(\vec{x},\vec{y})^j_k \;\; Q(\vec{y})^k_l \;\; U(\vec{x},\vec{y})^l_i \;\; , \tag{4.35}$$

where $P(\vec{x})$ and $Q(\vec{y})$ are the source and the sink of the gluelump. For simplicity, the points $\vec{x}$ and $\vec{y}$ are chosen on the same coordinate axis. In a 3D cubic lattice, for instance, a good overlap with the lowest 0^+ state is obtained by choosing P and Q as the 'clover leaves' [21] formed by the four plaquettes orthogonal to $\vec{y}-\vec{x}$. $U(\vec{x},\vec{y})$ is a shorthand notation for the parallel transporter formed by the product of the link variables in the fundamental representation along the straight path connecting the sites $\vec{x}$ and $\vec{y}$. Let us focus on the indices i and j of the operators U and $U^\dagger$. They belong to the reducible representation $\{N\}\otimes\{\bar{N}\}=\{N^2-1\}+\{1\}$. In order to project out the singlet and allow propagating the adjoint representation only, we perform the substitution

$$U^{\dagger}{}^j_k\, U^l_i \to U^{\dagger}{}^j_k\, U^l_i - \frac{1}{N}\,\delta^j_i\, U^{\dagger}{}^m_k\, U^l_m = U^{\dagger}{}^j_k\, U^l_i - \frac{1}{N}\,\delta^j_i\;\delta^l_k \;. \tag{4.36}$$

Inserting this projection in (4.35) yields

$$G(\vec{x},\vec{y}) = \mathrm{tr}\,(PUQU^\dagger) - \frac{1}{N}\,\mathrm{tr}\,P\;\mathrm{tr}\,Q\;. \tag{4.37}$$

In the case of $SU(2)$ we can use the trace identity

$$\mathrm{tr}\,A\,\mathrm{tr}\,B = \mathrm{tr}\,(A\,B) + \mathrm{tr}\,(A\,B^{-1})\;, \tag{4.38}$$

valid for any pair of 2×2 unimodular matrices [4], with $A=P$ and $B=UQU^\dagger$ and recast Eq.(4.37) in the form used by the lattice community

$$G(\vec{x},\vec{y}) = \frac{1}{2}\mathrm{tr}\,[PU(Q-Q^\dagger)U^\dagger] \equiv \frac{1}{2}\mathrm{tr}\,(P\sigma^a)\Gamma_{ab}\mathrm{tr}\,(Q\sigma^b)\;, \tag{4.39}$$

where the $\sigma^a\equiv\sigma_a$ are the Pauli matrices , $a=1,2,3$, and

$$\Gamma_{ab} = \frac{1}{2}\mathrm{tr}\,(\sigma_a U\sigma_b U^\dagger)\;. \tag{4.40}$$

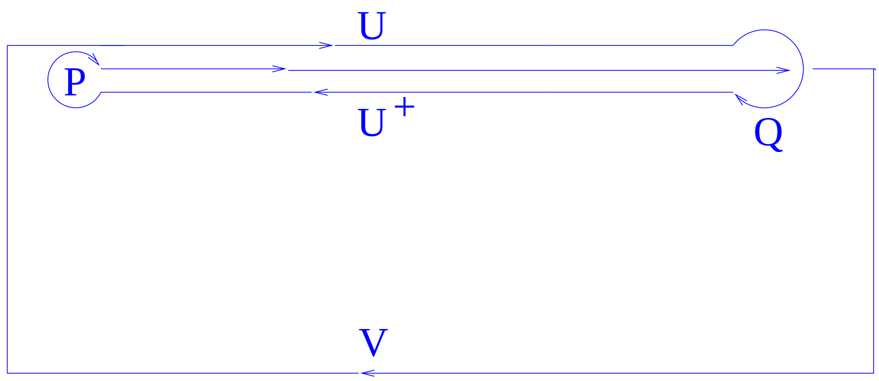

Figure 4: Mixed Wilson loop. The loop $(PUVUQU^\dagger)$ denotes a static source in the fundamental representation f. The three coincident lines $U, U, U^\dagger$ at the top are drawn separately for clarity. They belong to the reducible representation $f\otimes f\otimes \bar{f}$. Projection on a irreducible component is described in the text.

To construct a mixed Wilson loop let us start by considering an arbitrary closed path $\gamma = u\,v$ made by the composition of two open paths u and v. Let U and V be the group elements associated with these two paths respectively. The associated standard Wilson loop is

$$W(\gamma) = \mathrm{tr}\,(UV)\ . \tag{4.41}$$

where the trace is taken , here and in the following, in the fundamental representation f of the *non-abelian* gauge group G. We want to transform W in a mixed Wilson loop in which the source along the path u carries the quantum numbers of an higher representation $\mathcal{R}$ belonging to the same $N-$ ality class of f. In analogy with the construction of the gluelump operator, we start by considering the gauge-invariant generating operator (see Fig.4)

$$P_i^m\, U_l^i\, V_j^l\, U_k^j\, Q_n^k\, {U^\dagger}^n_m \tag{4.42}$$

where P and Q can be taken as the source operators associated to the "clovers" orthogonal to the path γ at the junctions separating u and v. Along the path u now propagates a source belonging to the reducible $f\otimes f\otimes \bar{f}$. We have then to project on some irreducible component.

To make a specific, illustrative example, let us consider the case of $SU(3)$, where we have

$$\{3\}\otimes\{3\}\otimes\{\bar{3}\} = 2\{3\}+\{\bar{6}\}+\{15\}\ . \tag{4.43}$$

We want to project on the $\{\bar{6}\}$ representation which has the same triality of $\{3\}$. It may be selected, for instance, by anti-symmetrizing the indices i and

[4] Actually it can be shown that the most general solution of the functional equation $\phi(A)\,\phi(B) = \phi(A\,B) + \phi(A\,B^{-1})$, where A and B are arbitrary elements of an unspecified group G and ϕ is a class function, is $G \equiv SL(2,\mathbb{C})$ and ϕ is the character of its fundamental representation. A study of this kind of relationships for various groups can be found in[43].

j and eliminating the traces with the index m in the matrix elements $P_i^m V_j^l$. Namely, we take the combination

$$\frac{3}{2}(P_j^m V_i^l - P_i^m V_j^l) + \frac{3}{4}[(\delta_i^m V_j^l - \delta_j^m V_i^l)\mathrm{tr}\, P - (VP)_j^l \delta_i^m + \delta_j^m (VP)_i^l] \quad (4.44)$$

and saturate with $U_l^i U_k^j Q_n^k U^{\dagger}{}^n_m$, getting finally

$$\begin{aligned} W_{\{3\}\to\{\bar{6}\}}(\gamma) = \frac{3}{2}\left[\mathrm{tr}\,(PUQU^\dagger)\,\mathrm{tr}\,(VU) - \mathrm{tr}\,(PUVUQU^\dagger)\right] + \\ \frac{3}{4}\left[\mathrm{tr}\, P\,\mathrm{tr}\,(VUQ) - \mathrm{tr}\,(VPUQ) + \mathrm{tr}\,(VPU)\,\mathrm{tr}\, Q - \mathrm{tr}\, P\mathrm{tr}\,(VU)\,\mathrm{tr}\, Q\,\right] \ . \end{aligned} \quad (4.45)$$

The fraction u of γ belonging to $\{\bar{6}\}$ is of course arbitrary. Moving the two junctions along γ we can manage as to shrink the length $|v|$ of v to zero, hence V is the identity matrix $V = \mathbb{I}$ and the whole loop carries the quantum numbers of $\{\bar{6}\}$. In this limit the two clovers associated to P and Q, which have opposite orientations, overlap, hence $P = Q^\dagger$. As a check of Eq.(4.45) we can now integrate over Q, using the standard orthogonality relations of the irreducible characters

$$\begin{aligned} \int_{Q\in G} dQ\, \mathrm{tr}_{\mathcal{R}}(Q\,A)\mathrm{tr}_{\mathcal{R}'}(Q^\dagger B) = \delta_{\mathcal{R},\mathcal{R}'}\,\frac{1}{d_{\mathcal{R}}}\mathrm{tr}_{\mathcal{R}}(A^\dagger B) \\ \int_{Q\in G} dQ\, \mathrm{tr}_{\mathcal{R}}(Q\,A\,Q^\dagger B) = \frac{1}{d_{\mathcal{R}}}\mathrm{tr}_{\mathcal{R}}(A^\dagger B)\ , \end{aligned} \quad (4.46)$$

written for any arbitrary compact group G. $\mathrm{tr}_{\mathcal{R}}$ is the character, *i.e.* the trace calculated in the irreducible representation $\mathcal{R}$ and $d_{\mathcal{R}} = \mathrm{tr}_{\mathcal{R}}(\mathbb{I})$ its dimension.

Putting in Eq.(4.45) $V = \mathbb{I}$, $P = Q^\dagger$ and integrating on Q yields

$$\frac{1}{2}\left[(\mathrm{tr}\, U)^2 - \mathrm{tr}\,(U^2)\right]\mathrm{tr}\, U^\dagger - \mathrm{tr}\, U = \frac{1}{2}\left[(\mathrm{tr}\, U^\dagger)^2 + \mathrm{tr}\,({U^\dagger}^2)\right] = \mathrm{tr}_{\{\bar{6}\}}(U)\ , \quad (4.47)$$

where the $SU(3)$ identity $\frac{1}{2}[(\mathrm{tr}\, U)^2 - \mathrm{tr}\,(U^2)] = \mathrm{tr}\, U^\dagger$ and its conjugate have been used.

Similarly, in the $SU(2)$ case, the mixed Wilson loop associated to the pair of representations $j = \frac{1}{2}$ and $j = \frac{3}{2}$ turns out to be

$$\begin{aligned} W_{\frac{1}{2}\to\frac{3}{2}}(\gamma) = \mathrm{tr}\,(PUVUQU^\dagger) + \mathrm{tr}\,(PUQU^\dagger)\mathrm{tr}\,(UV) - \\ \frac{1}{3}\left[\mathrm{tr}\,(PUQV) + \mathrm{tr}\,(PUV)\,\mathrm{tr}\, Q + \mathrm{tr}\, P\,(\mathrm{tr}\,(VUQ) + \mathrm{tr}\,(VU)\,\mathrm{tr}\, Q)\right]\ , \end{aligned} \quad (4.48)$$

where the normalisation is chosen, like in the $SU(3)$ case, in such a way that integration on Q after putting $V = \mathbb{I}$ and $P = Q^\dagger$ gives

$$\int_{Q\in SU(2)} dQ\, W_{\frac{1}{2}\to\frac{3}{2}} = \mathrm{tr}_{\frac{3}{2}}(U) \equiv \mathrm{tr}\,(U^2)\,\mathrm{tr}\, U\ . \quad (4.49)$$

It is clear that the above construction can be generalised to any non-abelian group and, in particular, to any fully anti-symmetric representation of $N-$ality k of $SU(N)$, which can be converted through the emission and the reabsorption of a glue-lump to an excited representation $\mathcal{R}$. It is also clear that one can build up mixed Wilson loops of the type drawn in Fig.1 c and d that we denote respectively as $W_{k\to\mathcal{R}}(r,t)$ and $W_{\mathcal{R}\to k\to\mathcal{R}}(r,t)$[5].

The static potential $V_{\mathcal{R}}(r)$ between the $\mathcal{R}$ sources and the decay of the associated unstable string into the stable $k-$string can then be extracted from measurements of the matrix correlator

$$C(r,t) = \begin{pmatrix} \langle W_{\mathcal{R}}(r,t)\rangle & \langle W_{k\to\mathcal{R}}(r,t)\rangle \\ \langle W_{k\to\mathcal{R}}(r,t)\rangle & \langle W_{\mathcal{R}\to k\to\mathcal{R}}(r,t)\rangle \end{pmatrix} \ , \tag{4.50}$$

where $W_{\mathcal{R}}(r,t)$ is the ordinary Wilson loop when the whole rectangle (r,t) is in the $\mathcal{R}$ representation. This is the generalisation of the multichannel method we alluded in the Introduction. Denoting by $\lambda(r,t)$ the highest eigenvalue of Eq.(4.50) we have

$$V_{\mathcal{R}}(r) = -\lim_{t\to\infty}\frac{1}{t}\log\lambda(r,t) \ , \tag{4.51}$$

which from a computational point of view is much better than the definition (3.24), because the mixed Wilson loops have, by construction, a much better overlap with the stable $k-$string state. In this way it will possible to evaluate the decay scale $r_{\mathcal{R}}$. One could also try to get a rough estimate of this quantity through Eq.(3.25). Indeed the instability of the $\mathcal{R}-$string leads to conjecture that the vacuum expectation value of $W_{k\to\mathcal{R}}(\gamma)$ should behave asymptotically as

$$\langle W_{k\to\mathcal{R}}(\gamma)\rangle \propto e^{-\mu_{\mathcal{R}}|v|-\mu'_{\mathcal{R}}|u|-\sigma_k A} \ , \tag{4.52}$$

where A is the area of the minimal surface encircled by γ and $|v|$ and $|u|$ are the lengths of the paths which carry the quantum numbers of the representations k and $\mathcal{R}$, respectively. This seems the most effective way to estimate the quantity $\mu'_{\mathcal{R}}$ and therefore $r_{\mathcal{R}}$.

5 Conclusion

In this paper we gained some insight into the physics of $SU(N)$ pure gauge theory by using in two different ways the standard decomposition of the direct product of irreducible representations of the gauge group.

First, we mapped this decomposition into the fusion rules of the effective CFT describing the critical behaviour of the finite-temperature deconfinement of those 2+1 dimensional gauge theories which undergo a continuous phase transition.

We worked out two specific examples, $SU(3)$ and $SU(4)$, which led us to conjecture the exact critical exponents to be attributed to Polyakov lines in some higher representations of the gauge group. The resulting functional form

[5] The "time" variable t is the vertical line in Fig.1.

of these Polyakov-Polyakov correlators at critically suggests generalising it to $T = 0$ Wilson loops in higher representations for any $SU(N)$. The proposed Ansatz offers a simple, general explanation of the difficulty to observe decaying unstable strings while measuring Wilson loops only.

The other way we exploited the mentioned decomposition in irreducible representations has been used to define a new gauge-invariant operator, the mixed Wilson loop $W_{\mathcal{R}\to\mathcal{S}}(\gamma)$, which describes a source belonging to an irreducible representation $\mathcal{R}$ which is converted into another representation $\mathcal{S}$ along part of its trajectory γ. Gauge invariance requires $\mathcal{R}$ and $\mathcal{S}$ having the same $N-$ality. This new operator could be used as a tool to study the decay of the unstable strings, a recurrent theme of this work.

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